

UNCLASSIFIED

AD 405 839

DEFENSE DOCUMENTATION CENTER

FOR

SCIENTIFIC AND TECHNICAL INFORMATION

CAMERON STATION, ALEXANDRIA, VIRGINIA



UNCLASSIFIED

NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

405839

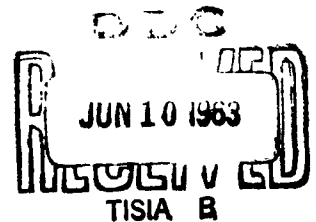
RADC-TDR-63-10, Suppl 1

Supplement to Third Quarterly Report

OPTIMUM APERTURE STUDY

Technical Documentary Report No. RADC-TDR-63-10, Suppl 1
May 1963

ROME AIR DEVELOPMENT CENTER
Research and Technology Division
Air Force Systems Command
United States Air Force
Griffiss Air Force Base
New York



Project No. 4506, Task No. 450604

(Prepared under Contract No. AF30(602)-2676
by D. Lee, Electronic Systems and Products
Division, Martin Company, Baltimore 3, Md.)

405839

63-3-5

Qualified requestors may obtain copies of this report from the ASTIA Document Service Center, Dayton 2, Ohio. ASTIA Services for the Department of Defense contractors are available through the "Field of Interest Register" on a "need-to-know" certified by the cognizant military agency of their project or contract.

RADC-TDR-63-10, Suppl 1

Supplement to Third Quarterly Report

OPTIMUM APERTURE STUDY

**Technical Documentary Report No. RADC-TDR-63-10, Suppl 1
May 1963**

**ROME AIR DEVELOPMENT CENTER
Research and Technology Division
Air Force Systems Command
United States Air Force
Griffiss Air Force Base
New York**

Project No. 4506, Task No. 450604

**(Prepared under Contract No. AF30(602)-2676
by D. Lee, Electronic Systems and Products
Division, Martin Company, Baltimore 3, Md.)**

CONTENTS

	Page
Abstract	v
I. Introduction	1
II. Elliptical Aperture with Nonoptimum Illumination	3
III. Graphs	31
IV. Table of Comparison	37
Appendix	39

ABSTRACT

The object of this contract is to study the applicability of the Wiener-Spencer Theorem to antennas. This theorem states that minimum standard deviation of the far-field pattern occurs when the illumination function corresponds to the lowest mode of vibration of a membrane stretched across the aperture opening.


This report presents the investigation of four selected nonoptimum illuminations for the elliptical apertures. Approximations are used to obtain expressions for far-field power patterns, and second moments are tabulated. In addition, illuminations and far-field power patterns are plotted.

Title of Report RADC-TDR-63-10, Suppl 1

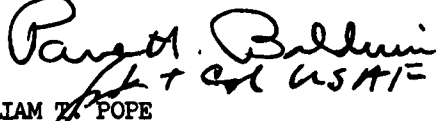
PUBLICATION REVIEW

This report has been reviewed and is approved.

Approved:


ARTHUR J. FROHLICH
Chief, Techniques Laboratory
Directorate of Aerospace Surveillance & Control

Approved:


WILLIAM T. POPE
Acting Director
Director of Aerospace
Surveillance & Control

I. INTRODUCTION

The Third Quarterly Report states that a second group of nonoptimum illuminations for elliptical apertures of the form

$$F(\xi, \eta) = \left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N \quad N = 1, 2, 3, 4$$

will be investigated. It further states that a comparison will be made between the optimum and nonoptimum illuminations. The following work has been accomplished:

- (1) The far-field power patterns of elliptical apertures with four selected nonoptimum illuminations were derived through approximation.
- (2) An IBM 1620 program was written to tabulate the moments.
- (3) Investigation was made between optimum and nonoptimum illuminations to the degree of improvement in terms of the second moments, the side lobes and the beamwidth.
- (4) Far-field power patterns of four selected nonoptimum illuminations were plotted along major axes.

II. ELLIPTICAL APERTURE WITH NONOPTIMUM ILLUMINATION

The far-field voltage power pattern of elliptical aperture is given by

$$G(u, v) = \iint e^{i(ux + vy)} F \, dx dy.$$

For the optimum case, F is a product of two Mathieu Functions

$$[Ce(q, \xi)] [ce(q, \eta)]$$

In elliptical coordinates,

$$G(u, v) = \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} e^{ih[u \cosh \xi \cos \eta + v \sinh \xi \sin \eta]} (\cosh 2\xi - \cos 2\eta) F(\xi, \eta) \, d\xi d\eta$$

where $F(\xi, \eta)$ is the illumination distribution.

The zeroth moment is given by

$$\mu_0 = \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} F^2(\xi, \eta) (\cosh 2\xi - \cos 2\eta) \, d\xi d\eta,$$

and the second moment is given by

$$\mu_2 = \int_0^{2\pi} \int_0^{\xi_0} \left[\left(\frac{\partial F}{\partial \xi} \right)^2 + \left(\frac{\partial F}{\partial \eta} \right)^2 \right] d\xi d\eta.$$

For a nonoptimum illumination, let

$$F(\xi, \eta) = \left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N$$

The illumination satisfies the conditions

$$F(\xi, \eta) = F(\xi, \eta + 2\pi)$$

$$F(\xi_0, \eta) = 0.$$

Thus,

$$\begin{aligned} G(u, v) &= \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} e^{ih} [u \cosh \xi \cos \eta + v \sinh \xi \sin \eta] (\cosh 2\xi \\ &\quad - \cos 2\eta) \left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N d\xi d\eta \\ &= \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} e^{ih} [u \cosh \xi \cos \eta + v \sinh \xi \sin \eta] \cosh 2\xi \left[(1 \right. \\ &\quad \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N d\xi d\eta \\ &\quad - \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} e^{ih} [u \cosh \xi \cos \eta + v \sinh \xi \sin \eta] \cos 2\eta \left[(1 \right. \\ &\quad \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N d\xi d\eta. \end{aligned}$$

The preceding integrals do not appear to be solvable in closed form. Instead, we examine $G(u, 0)$.

$$\begin{aligned} G(u, 0) &= \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} e^{ih} u \cosh \xi \cos \eta \cosh 2\xi \left[(1 \right. \\ &\quad \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N d\xi d\eta \\ &\quad - \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} e^{ih} u \cosh \xi \cos \eta \cos 2\eta \left[(1 \right. \\ &\quad \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N d\xi d\eta. \end{aligned}$$

For the given aperture $\xi_0 = 0.277$, $0 \leq \xi \leq \xi_0$

$$\cos \xi \sim 1$$

$$\sinh \xi \sim \xi.$$

Thus,

$$\begin{aligned} G(u, 0) = & \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} e^{ih u \cos \eta} \cosh 2\xi \left[(1 \right. \\ & \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N d\xi d\eta \\ & - \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} e^{ih u \cos \eta} \cos 2\xi \left[(1 \right. \\ & \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N d\xi d\eta. \end{aligned}$$

Recall that

$$\begin{aligned} e^{ix \cos \theta} = & J_0(x) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(x) \cos 2K \theta \\ & + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(x) \cos (2K-1) \theta. \end{aligned}$$

Thus,

$$\begin{aligned} G(u, 0) = & \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \right. \\ & \left. + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \right] \cosh 2\xi \left[(1 \right. \\ & \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N d\xi d\eta \end{aligned}$$

$$\begin{aligned}
& -\frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \right. \\
& + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \left. \right] \cos 2\eta \left[(1 \right. \\
& \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N d\xi d\eta.
\end{aligned}$$

For $N=1$,

$$\begin{aligned}
G_1(u, 0) &= \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \right. \\
& + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \left. \right] \cosh 2\xi_0 \left[(1 \right. \\
& \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right] d\xi d\eta \\
& - \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \right. \\
& + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \left. \right] \cos 2\eta \left[(1 \right. \\
& \left. + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right] d\xi d\eta
\end{aligned}$$

$$\begin{aligned}
&= \frac{h^2}{2} \left(\int_0^{2\pi} J_0(hu) (1 + a \sin^2 \eta) d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos \frac{\pi \xi}{2\xi_0} d\xi \right) \\
&+ \frac{h^2}{2} \left(\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta (1 \right. \\
&\quad \left. + a \sin^2 \eta) d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos \frac{\pi \xi}{2\xi_0} d\xi \right) \\
&+ \frac{h^2}{2} \left(\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta (1 \right. \\
&\quad \left. + a \sin^2 \eta) d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos \frac{\pi \xi}{2\xi_0} d\xi \right) \\
&- \frac{h^2}{2} \left(\int_0^{2\pi} J_0(hu) \cos 2 \eta \left[(1 + a \sin^2 \eta) \right] d\eta \right) \left(\int_0^{\xi_0} \cos \frac{\pi \xi}{2\xi_0} d\xi \right) \\
&- \frac{h^2}{2} \left(\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \cos 2 \eta (1 \right. \\
&\quad \left. + a \sin^2 \eta) d\eta \right) \left(\int_0^{\xi_0} \cos \frac{\pi \xi}{2\xi_0} d\xi \right) \\
&- \frac{h^2}{2} \left(\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \cos 2 \eta (1 \right.
\end{aligned}$$

$$+ a \sin^2 \eta) d\eta) \left(\int_0^{\xi_0} \cos \frac{\pi \xi}{2\xi_0} d\xi \right).$$

Here,

$$\int_0^{\xi_0} \cosh 2\xi \cos \frac{\pi \xi}{2\xi_0} d\xi = \frac{2\xi_0 \pi \cosh 2\xi_0}{16 \xi_0^2 + \pi^2}$$

$$\int_0^{2\pi} J_0(hu) (1 + a \sin^2 \eta) d\eta = 2\pi \left(1 + \frac{a}{2}\right) J_0(hu)$$

$$\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta (1 + a \sin^2 \eta) d\eta = a \pi J_2(hu)$$

$$\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta (1 + a \sin^2 \eta) d\eta = 0$$

$$\int_0^{\xi_0} \cos \frac{\pi \xi}{2\xi_0} d\xi = \frac{2\xi_0}{\pi}$$

$$\int_0^{2\pi} J_0(hu) \cos 2\eta \left[(1 + a \sin^2 \eta) \right] d\eta = -\frac{a}{2} \pi J_0(hu)$$

$$\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \cos 2\eta \left[(1 + a \sin^2 \eta) \right] d\eta$$

$$= -2\pi \left(1 + \frac{a}{2}\right) J_2(hu) - \frac{a}{2} J_4(hu)$$

$$\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos(2K-1)\eta \cos 2\eta \left[(1 + a \sin^2 \eta) \right] d\eta = 0.$$

Therefore,

$$G_1(u, 0) = h^2 \xi_0 \left[\left\{ \frac{a}{2} + \left(1 + \frac{a}{2}\right) \frac{2\pi^2 \cosh 2\xi_0}{16\xi_0^2 + \pi^2} \right\} J_0(hu) + \left\{ 2\left(1 + \frac{a}{2}\right) + \frac{a\pi^2 \cosh 2\xi_0}{16\xi_0^2 + \pi^2} \right\} J_2(hu) + \frac{a}{2} J_4(hu) \right]$$

For $N=2$,

$$\begin{aligned} G_2(u, 0) = & \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K\eta \right. \\ & \left. + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos(2K-1)\eta \right] \cdot \cosh 2\xi \left[(1 + a \sin^2 \eta) \cos \frac{\pi\xi}{2\xi_0} \right]^2 d\xi d\eta \\ & - \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K\eta \right. \\ & \left. + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos(2K-1)\eta \right] \cdot \cos 2\eta \left[(1 \right. \end{aligned}$$

$$\begin{aligned}
& + a \sin^2 \eta \cos \frac{\pi \xi}{2\xi_0} \Big]^2 d\xi d\eta \\
& = \frac{h^2}{2} \left(\int_0^{2\pi} J_0(hu) (1 + a \sin^2 \eta)^2 d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^2 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& + \frac{h^2}{2} \left(\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta (1 \right. \\
& \left. + a \sin^2 \eta)^2 d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^2 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& + \frac{h^2}{2} \left(\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta (1 \right. \\
& \left. + a \sin^2 \eta)^2 d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^2 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& - \frac{h^2}{2} \left(\int_0^{2\pi} J_0(hu) \cos 2\eta \left[(1 + a \sin^2 \eta) \right]^2 d\eta \right) \left(\int_0^{\xi_0} \cos^2 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& - \frac{h^2}{2} \left(\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \cos 2\eta \left[(1 \right. \right. \\
& \left. \left. + a \sin^2 \eta) \right]^2 d\eta \right) \left(\int_0^{\xi_0} \cos^2 \frac{\pi \xi}{2\xi_0} d\xi \right)
\end{aligned}$$

$$- \frac{h^2}{2} \left(\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos(2K-1)\eta \cos 2\eta \left[1 + a \sin^2 \eta \right]^2 d\eta \right) \left(\int_0^{\xi_0} \cos^2 \frac{\pi \xi}{2\xi_0} d\xi \right).$$

Here,

$$\int_0^{\xi_0} \cosh 2\xi \cos^2 \frac{\pi \xi}{2\xi_0} d\xi = \frac{\pi^2 \sinh 2\xi_0}{4(4\xi_0^2 + \pi^2)}$$

$$\int_0^{2\pi} J_0(hu) (1 + a \sin^2 \eta)^2 d\eta = 2\pi \left(1 + a + \frac{3}{8} a^2 \right) J_0(hu)$$

$$\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K\eta (1 + a \sin^2 \eta)^2 d\eta = 2\pi a \left(1 + \frac{a}{2} \right) J_2(hu) + \frac{a^2}{4} \pi J_4(hu)$$

$$\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos(2K-1)\eta (1 + a \sin^2 \eta)^2 d\eta = 0$$

$$\int_0^{\xi_0} \cos^2 \frac{\pi \xi}{2\xi_0} d\xi = \frac{\xi_0}{2}$$

$$\int_0^{2\pi} J_0(hu) \cos 2\eta (1 + a \sin^2 \eta)^2 d\eta = -a \left(1 + \frac{a}{2} \right) \pi J_0(hu)$$

$$\begin{aligned}
& \int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \cos 2 \eta (1 + a \sin^2 \eta)^2 d\eta \\
& = - \left(2 + 2a + \frac{7}{8} a^2 \right) \pi J_2(hu) - a \left(1 + \frac{a}{2} \right) \pi J_4(hu) - \frac{a^2}{8} \pi J_6(hu) \\
& \int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \cos 2 \eta (1 \\
& + a \sin^2 \eta)^2 d\eta = 0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
G_2(u, 0) = h^2 \pi & \left[\left\{ \left(1 + a + \frac{3}{8} a^2 \right) \frac{\pi^2 \sinh 2 \xi_0}{4 (4 \xi_0^2 + \pi^2)} + \frac{\xi_0 a}{4} \left(1 \right. \right. \right. \\
& \left. \left. + \frac{a}{2} \right) \right\} J_0(hu) + \left\{ a \left(1 + \frac{a}{2} \right) \frac{\pi^2 \sinh 2 \xi_0}{4 (4 \xi_0^2 + \pi^2)} + \frac{\xi_0}{4} \left(2 \right. \right. \\
& \left. \left. + 2a + \frac{7}{8} a^2 \right) \right\} J_2(hu) + \left\{ \frac{a^2}{8} \cdot \frac{\pi^2 \sinh 2 \xi_0}{4 (4 \xi_0^2 + \pi^2)} \right. \\
& \left. \left. + \frac{\xi_0 a}{4} \left(1 + \frac{a}{2} \right) \right\} J_4(hu) + \frac{\xi_0 a^2}{32} J_6(hu) \right].
\end{aligned}$$

For $N = 3$,

$$\begin{aligned}
G_3(u, 0) = \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} & \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \right. \\
& \left. + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \right] \cdot \cosh 2\xi \left[\left(1 \right. \right. \\
& \left. \left. + a \sin^2 \eta \cos \frac{\pi \xi}{2 \xi_0} \right) \right]^3 d\xi d\eta
\end{aligned}$$

$$\begin{aligned}
& - \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \right. \\
& + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \left. \right] \cdot \cos 2 \eta \left[(1 \right. \\
& + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \left. \right]^3 d\xi d\eta \\
& = \frac{h^2}{2} \left(\int_0^{2\pi} J_0(hu) (1 + a \sin^2 \eta)^3 d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^3 \left(\frac{\pi \xi}{2\xi_0} \right) d\xi \right) \\
& + \frac{h^2}{2} \left(\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta (1 \right. \\
& + a \sin^2 \eta)^3 d\eta \left. \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^3 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& + \frac{h^2}{2} \left(\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta (1 \right. \\
& + a \sin^2 \eta)^3 d\eta \left. \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^3 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& - \frac{h^2}{2} \left(\int_0^{2\pi} J_0(hu) \cos 2 \eta (1 + a \sin^2 \eta)^3 d\eta \right) \left(\int_0^{\xi_0} \cos^3 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& - \frac{h^2}{2} \left(\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \cos 2 \eta (1 \right. \\
& + a \sin^2 \eta)^3 d\eta \left. \right) \left(\int_0^{\xi_0} \cos^3 \frac{\pi \xi}{2\xi_0} d\xi \right)
\end{aligned}$$

$$- \frac{h^2}{2} \left(\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos(2K-1)\eta \cos 2\eta (1 + a \sin^2 \eta)^3 d\eta \right) \left(\int_0^{\xi_0} \cosh^3 \frac{\pi \xi}{2\xi_0} d\xi \right).$$

Here,

$$\int_0^{\xi_0} \cosh 2\xi \cos^3 \frac{\pi \xi}{2\xi_0} d\xi = \frac{3}{2} \xi_0 \pi \cosh 2\xi_0 \left[\frac{1}{16 \xi_0^2 + \pi^2} - \frac{1}{16 \xi_0^2 + 9 \pi^2} \right]$$

$$\int_0^{2\pi} J_0(hu) (1 + a \sin^2 \eta)^3 d\eta = \left(1 + \frac{3}{2} a + \frac{9}{8} a^2 + \frac{5}{16} a^3 \right) 2\pi J_0(hu)$$

$$\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K\eta (1 + a \sin^2 \eta)^3 d\eta =$$

$$\left(\frac{3}{2} a + \frac{3}{2} a^2 + \frac{15}{32} a^3 \right) 2\pi J_2(hu) + \frac{3}{4} a^2 \left(1 + \frac{a}{2} \right) \pi J_4(hu)$$

$$+ \frac{1}{16} a^3 \pi J_6(hu)$$

$$\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos(2K-1)\eta (1 + a \sin^2 \eta)^3 d\eta = 0$$

$$\int_0^{\xi_0} \cos^3 \frac{\pi \xi}{2\xi_0} d\xi = \frac{4\xi_0}{3\pi}$$

$$\int_0^{2\pi} J_0(hu) \cos 2\eta (1 + a \sin^2 \eta)^3 d\eta = - \left(\frac{3}{2} a + \frac{3}{2} a^2 + \frac{15}{32} a^3 \right) \pi J_0(hu)$$

$$\begin{aligned} \int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K\eta \cos 2\eta (1 + a \sin^2 \eta)^3 d\eta = \\ - \left(2 + 3a + \frac{21}{8} a^2 + \frac{13}{16} a^3 \right) \pi J_2(hu) - a \left(\frac{3}{2} + \frac{3}{2} a + \frac{1}{2} a^2 \right) \pi J_4(hu) \\ - \frac{3}{8} a^2 \left(1 + \frac{a}{2} \right) \pi J_6(hu) - \frac{1}{32} a^3 \pi J_8(hu) \end{aligned}$$

$$\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1)\eta \cos 2\eta (1 + a \sin^2 \eta)^3 d\eta = 0.$$

Therefore,

$$G_3(u, 0) = h^2 \xi_0 \sum_{r=0}^4 \alpha_{2r} J_{2r}(hu)$$

where

$$\begin{aligned} \alpha_0 = \frac{3}{2} \pi^2 \cosh 2\xi_0 \left[\frac{1}{16 \xi_0^2 + \pi^2} - \frac{1}{16 \xi_0^2 + 9\pi^2} \right] \left(1 \right. \\ \left. + \frac{3}{2} a + \frac{9}{8} a^2 + \frac{5}{16} a^3 \right) + \frac{2}{3} a \left(\frac{3}{2} + \frac{3}{2} a + \frac{15}{32} a^2 \right) \end{aligned}$$

$$\begin{aligned}
\alpha_2 &= \frac{3}{2} \pi^2 \cosh 2\xi_0 \left[\frac{1}{16 \xi_0^2 + \pi^2} - \frac{1}{16 \xi_0^2 + 9 \pi^2} \right] \left(\frac{3}{2} a + \frac{3}{2} a^2 \right. \\
&\quad \left. + \frac{15}{32} a^3 \right) + \frac{2}{3} \left(2 + 3a + \frac{21}{8} a^2 + \frac{13}{16} a^3 \right) \\
\alpha_4 &= \frac{9}{16} \pi^2 \cosh 2\xi_0 \left[\frac{1}{16 \xi_0^2 + \pi^2} - \frac{1}{16 \xi_0^2 + 9 \pi^2} \right] a^2 \left(1 + \frac{a}{2} \right) \\
&\quad + \frac{2}{3} a \left(\frac{3}{2} + \frac{3}{2} a + \frac{1}{2} a^2 \right) \\
\alpha_6 &= \frac{3}{64} \pi^2 \cosh 2\xi_0 \left[\frac{1}{16 \xi_0^2 + \pi^2} - \frac{1}{16 \xi_0^2 + 9 \pi^2} \right] a^3 \\
&\quad + \frac{1}{4} a^2 \left(1 + \frac{a}{2} \right) \\
\alpha_8 &= \frac{1}{48} a^3.
\end{aligned}$$

For $N = 4$,

$$\begin{aligned}
G_4(u, 0) &= \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \right. \\
&\quad \left. + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \right] \cdot \cosh 2\xi \left[1 \right. \\
&\quad \left. + a \sin^2 \eta \cos \frac{\pi \xi}{2\xi_0} \right]^4 d\xi d\eta
\end{aligned}$$

$$\begin{aligned}
& - \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \left[J_0(hu) + 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K\eta \right. \\
& + 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1)\eta \left. \right] \cos 2\eta \left[(1 \right. \\
& + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \left. \right]^4 d\xi d\eta \\
& = \frac{h^2}{2} \left(\int_0^{2\pi} J_0(hu) (1 + a \sin^2 \eta)^4 d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^4 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& + \frac{h^2}{2} \left(\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K\eta (1 \right. \\
& + a \sin^2 \eta)^4 d\eta \left. \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^4 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& + \frac{h^2}{2} \left(\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1)\eta (1 \right. \\
& + a \sin^2 \eta)^4 d\eta \left. \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^4 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& - \frac{h^2}{2} \left(\int_0^{2\pi} J_0(hu) \cos 2\eta (1 + a \sin^2 \eta)^4 d\eta \right) \left(\int_0^{\xi_0} \cos^4 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
& - \frac{h^2}{2} \left(\int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K\eta \cos 2\eta (1 \right. \\
& + a \sin^2 \eta)^4 d\eta \left. \right) \left(\int_0^{\xi_0} \cos^4 \frac{\pi \xi}{2\xi_0} d\xi \right)
\end{aligned}$$

$$- \frac{h^2}{2} \left(\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos(2K-1)\eta \cos 2\eta (1 + a \sin^2 \eta)^4 d\eta \right) \left(\int_0^{\xi_0} \cos^4 \frac{\pi \xi}{2\xi_0} d\xi \right).$$

Here,

$$\int_0^{\xi_0} \cosh 2\xi \cos^4 \frac{\pi \xi}{2\xi_0} d\xi = \left(\frac{3}{16} - \frac{\xi_0^2}{4 \xi_0^2 + \pi^2} + \frac{\xi_0^2}{16 (\xi_0^2 + \pi^2)} \right) \sinh 2\xi_0$$

$$\int_0^{2\pi} J_0(hu) (1 + a \sin^2 \eta)^4 d\eta = 2\pi \left(1 + 2a + \frac{9}{4} a^2 + \frac{5}{4} a^3 + \frac{35}{128} a^4 \right) J_0(hu)$$

$$\begin{aligned} \int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K\eta (1 + a \sin^2 \eta)^4 d\eta &= 2\pi \left(2a \right. \\ &+ 3a^2 + \frac{15}{8} a^3 + \frac{7}{16} a^4 \left. \right) J_2(hu) + 2\pi \left(\frac{3}{4} a^2 + \frac{3}{4} a^3 + \frac{7}{32} a^4 \right) J_4(hu) \\ &+ 2\pi \left(\frac{1}{8} a^3 + \frac{1}{16} a^4 \right) J_6(hu) + 2\pi \frac{1}{128} a^4 J_8(hu) \end{aligned}$$

$$\int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos(2K-1)\eta (1 + a \sin^2 \eta)^4 d\eta = 0$$

$$\int_0^{\xi_0} \cos^4 \frac{\pi \xi}{2\xi_0} d\xi = \frac{3}{8} \xi_0$$

$$\int_0^{2\pi} J_0(hu) \cos 2\eta (1 + a \sin^2 \eta)^4 d\eta = - \left(2a + 3a^2 + \frac{15}{8} a^3 + \frac{7}{16} a^4 \right) \pi J_0(hu)$$

$$\begin{aligned} \int_0^{2\pi} 2 \sum_{K=1}^{\infty} (-)^K J_{2K}(hu) \cos 2K \eta \cos 2\eta (1 + a \sin^2 \eta)^4 d\eta = & - \left(2a + 4a + \frac{21}{4} a^2 + \frac{13}{4} a^3 + \frac{49}{64} a^4 \right) \pi J_2(hu) - \left(2a + 3a^2 + 2a^3 + \frac{1}{2} a^4 \right) \pi J_4(hu) \\ & - \left(\frac{3}{4} a^2 + \frac{3}{4} a^3 + \frac{29}{128} a^4 \right) \pi J_6(hu) \\ & - \left(\frac{1}{8} a^3 + \frac{1}{16} a^4 \right) \pi J_8(hu) - \frac{1}{128} a^4 \pi J_{10}(hu) \end{aligned}$$

$$\begin{aligned} \int_0^{2\pi} 2i \sum_{K=1}^{\infty} (-)^{K-1} J_{2K-1}(hu) \cos (2K-1) \eta \cos 2\eta (1 + a \sin^2 \eta)^4 d\eta = 0. \end{aligned}$$

Therefore,

$$G_4(u, 0) = h^2 \pi \sum_{r=0}^5 \beta_{2r} J_{2r}(hu)$$

where

$$\begin{aligned} \beta_0 = & \left(1 + 2a + \frac{9}{4} a^2 + \frac{5}{4} a^3 + \frac{35}{128} a^4 \right) \left(\frac{3}{16} - \frac{\xi_0^2}{4 \xi_0^2 + \pi^2} + \frac{\xi_0^2}{16 (\xi_0^2 + \pi^2)} \right) \sinh 2\xi_0 + \frac{3}{16} \xi_0 \left(2a + 3a^2 + \frac{15}{8} a^3 + \frac{7}{16} a^4 \right) \end{aligned}$$

$$\begin{aligned}\beta_2 = & \left(2a + 3a^2 + \frac{15}{8} a^3 + \frac{7}{16} a^4\right) \left(\frac{3}{16} - \frac{\xi_0^2}{4\xi_0^2 + \pi^2}\right. \\ & \left. + \frac{\xi_0^2}{16(\xi_0^2 + \pi^2)}\right) \sinh 2\xi_0 + \frac{3}{16} \xi_0 \left(2 + 4a + \frac{21}{4} a^2\right. \\ & \left. + \frac{13}{4} a^3 + \frac{49}{64} a^4\right)\end{aligned}$$

$$\begin{aligned}\beta_4 = & \left(\frac{3}{4} a^2 + \frac{3}{4} a^3 + \frac{7}{32} a^4\right) \left(\frac{3}{16} - \frac{\xi_0^2}{4\xi_0^2 + \pi^2} + \frac{\xi_0^2}{16(\xi_0^2 + \pi^2)}\right) \sinh 2\xi_0 \\ & + \frac{3}{16} \xi_0 \left(2a + 3a^2 + 2a^3 + \frac{1}{2} a^4\right)\end{aligned}$$

$$\begin{aligned}\beta_6 = & \frac{1}{8} a^3 \left(1 + \frac{1}{2} a\right) \left(\frac{3}{16} - \frac{\xi_0^2}{4\xi_0^2 + \pi^2} + \frac{\xi_0^2}{16(\xi_0^2 + \pi^2)}\right) \sinh 2\xi_0 \\ & + \frac{3}{16} \xi_0 \left(\frac{3}{4} a^2 + \frac{3}{4} a^3 + \frac{29}{128} a^4\right)\end{aligned}$$

$$\begin{aligned}\beta_8 = & \frac{1}{128} a^4 \left(\frac{3}{16} - \frac{\xi_0^2}{4\xi_0^2 + \pi^2} + \frac{\xi_0^2}{16(\xi_0^2 + \pi^2)}\right) \sinh 2\xi_0 \\ & + \frac{3}{128} \xi_0 a^3 \left(1 + \frac{1}{2} a\right)\end{aligned}$$

$$\beta_{10} = \frac{3}{2048} \xi_0 a^4.$$

For the zeroth moment in elliptical coordinates,

$$\begin{aligned}
 \mu_0 &= \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} (\cosh 2\xi - \cos 2\eta) F^2(\xi, \eta) d\xi d\eta \\
 &= \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} (\cosh 2\xi - \cos 2\eta) \left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^{2N} d\xi d\eta \\
 &= \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \cosh 2\xi \left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^{2N} d\xi d\eta \\
 &\quad - \frac{h^2}{2} \int_0^{2\pi} \int_0^{\xi_0} \cos 2\eta \left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^{2N} d\xi d\eta.
 \end{aligned}$$

For $N = 1$,

$$\begin{aligned}
 \mu_{0,1} &= \frac{h^2}{2} \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^2 d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^2 \frac{\pi \xi}{2\xi_0} d\xi \right) \\
 &\quad - \frac{h^2}{2} \left(\int_0^{2\pi} \cos 2\eta (1 + a \sin^2 \eta)^2 d\eta \right) \left(\int_0^{\xi_0} \cos^2 \frac{\pi \xi}{2\xi_0} d\xi \right).
 \end{aligned}$$

Here,

$$\begin{aligned}
 \int_0^{2\pi} (1 + a \sin^2 \eta)^2 d\eta &= 2\pi \left(1 + a + \frac{3}{8} a^2 \right) \\
 \int_0^{\xi_0} \cosh 2\xi \cos^2 \frac{\pi \xi}{2\xi_0} d\xi &= \frac{\pi^2 \sinh 2\xi_0}{4(4\xi_0^2 + \pi^2)}
 \end{aligned}$$

$$\int_0^{2\pi} \cos 2\eta (1 + a \sin^2 \eta)^2 d\eta = -a \left(1 + \frac{a}{2}\right) \pi$$

$$\int_0^{\xi_0} \cos^2 \frac{\pi \xi}{2\xi_0} d\xi = \frac{1}{2} \xi_0.$$

Therefore,

$$\mu_{0,1} = h^2 \pi \left[\left(1 + a + \frac{3}{8} a^2\right) \frac{\pi^2 \sinh 2\xi_0}{4(4\xi_0^2 + \pi^2)} + \frac{1}{4} \xi_0 a \left(1 + \frac{1}{2} a\right) \right]. \quad (1)$$

For $N = 2$,

$$\begin{aligned} \mu_{0,2} = & \frac{h^2}{2} \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^4 d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^4 \frac{\pi \xi}{2\xi_0} d\xi \right) \\ & - \frac{h^2}{2} \left(\int_0^{2\pi} \cos 2\eta (1 + a \sin^2 \eta)^4 d\eta \right) \left(\int_0^{\xi_0} \cos^4 \left(\frac{\pi \xi}{2\xi_0} \right) d\xi \right) \end{aligned}$$

Here,

$$\int_0^{2\pi} (1 + a \sin^2 \eta)^4 d\eta = \left(1 + 2a + \frac{9}{4} a^2 + \frac{5}{4} a^3 + \frac{35}{128} a^4\right) 2\pi$$

$$\int_0^{\xi_0} \cosh 2\xi \cos^4 \frac{\pi \xi}{2\xi_0} d\xi = \left(\frac{3}{16} - \frac{\xi_0^2}{4\xi_0^2 + \pi^2} + \frac{\xi_0^2}{16(\xi_0^2 + \pi^2)} \right) \sinh 2\xi_0$$

$$\int_0^{2\pi} \cos 2\eta (1 + a \sin^2 \eta)^4 d\eta = - \left(2a + 3a^2 + \frac{15}{8} a^3 + \frac{7}{16} a^4 \right) \pi$$

$$\int_0^{\xi_0} \cos^4 \frac{\pi \xi}{2\xi_0} d\xi = \frac{3}{8} \xi_0.$$

Therefore,

$$\begin{aligned} \mu_{0,2} = h^2 \pi \left[\left(1 + 2a + \frac{9}{4} a^2 + \frac{5}{4} a^3 + \frac{35}{128} a^4 \right) \left(\frac{3}{16} \right. \right. \\ \left. \left. - \frac{\xi_0^2}{4 \xi_0^2 + \pi^2} + \frac{\xi_0^2}{16 (\xi_0^2 + \pi^2)} \right) \sinh 2 \xi_0 \right. \\ \left. + \frac{3}{16} \xi_0 \left(2a + 3a^2 + \frac{15}{8} a^3 + \frac{7}{16} a^4 \right) \right]. \end{aligned} \quad (2)$$

For $N = 3$,

$$\begin{aligned} \mu_{0,3} = \frac{h^2}{2} \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^6 d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^6 \frac{\pi \xi}{2\xi_0} d\xi \right) \\ - \frac{h^2}{2} \left(\int_0^{2\pi} \cos 2\eta (1 + a \sin^2 \eta)^6 d\eta \right) \left(\int_0^{\xi_0} \cos^6 \frac{\pi \xi}{2\xi_0} d\xi \right). \end{aligned}$$

Here,

$$\begin{aligned} \int_0^{2\pi} (1 + a \sin^2 \eta)^6 d\eta = 2\pi \left(1 + 3a + \frac{45}{8} a^2 + \frac{25}{4} a^3 + \frac{525}{128} a^4 \right. \\ \left. + \frac{189}{128} a^5 + \frac{231}{1024} a^6 \right) \\ \int_0^{\xi_0} \cosh 2\xi \cos^6 \frac{\pi \xi}{2\xi_0} d\xi = \left(\frac{5}{32} - \frac{15 \xi_0^2}{16 (4 \xi_0^2 + \pi^2)} + \frac{3 \xi_0^2}{32 (\xi_0^2 + \pi^2)} \right. \\ \left. - \frac{\xi_0^2}{16 (\xi_0^2 + 9 \pi^2)} \right) \sinh 2 \xi_0 \end{aligned}$$

$$\int_0^{2\pi} \cos 2\eta (1 + a \sin^2 \eta)^6 d\eta = -\pi a \left(3 + \frac{15}{2} a + \frac{75}{8} a^2 \right. \\ \left. + \frac{105}{16} a^3 + \frac{315}{128} a^4 + \frac{99}{256} a^5 \right) \\ \int_0^{\xi_0} \cos^6 \frac{\pi \xi}{2\xi_0} d\xi = \frac{5}{16} \xi_0.$$

Therefore,

$$\mu_{0,3} = \frac{1}{8} h^2 \pi \left[\left(1 + 3a + \frac{45}{8} a^2 + \frac{25}{4} a^3 + \frac{525}{128} a^4 + \frac{189}{128} a^5 \right. \right. \\ \left. \left. + \frac{231}{1024} a^6 \right) \left(\frac{5}{4} - \frac{15 \xi_0^2}{2(4\xi_0^2 + \pi^2)} + \frac{3 \xi_0^2}{4(\xi_0^2 + \pi^2)} \right. \right. \\ \left. \left. - \frac{\xi_0^2}{2(4\xi_0^2 + 9\pi^2)} \right) \sinh 2\xi_0 + \frac{5}{4} \xi_0 a \left(3 + \frac{15}{2} a + \frac{75}{8} a^2 \right. \right. \\ \left. \left. + \frac{105}{16} a^3 + \frac{315}{128} a^4 + \frac{99}{256} a^5 \right) \right]. \quad (3)$$

For $N = 4$,

$$\mu_{0,4} = \frac{h^2}{2} \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^8 d\eta \right) \left(\int_0^{\xi_0} \cosh 2\xi \cos^8 \frac{\pi \xi}{2\xi_0} d\xi \right) \\ - \frac{h^2}{2} \left(\int_0^{2\pi} \cos 2\eta (1 + a \sin^2 \eta)^8 d\eta \right) \left(\int_0^{\xi_0} \cos^8 \frac{\pi \xi}{2\xi_0} d\xi \right).$$

Here,

$$\begin{aligned}
 \int_0^{2\pi} (1 + a \sin^2 \eta)^8 d\eta &= 2\pi \left(1 + 4a + \frac{21}{2} a^2 + \frac{35}{2} a^3 + \frac{1225}{64} a^4 \right. \\
 &\quad \left. + \frac{441}{32} a^5 + \frac{1617}{128} a^6 + \frac{429}{128} a^7 + \frac{6335}{32768} a^8 \right) \\
 \int_0^{\xi_0} \cosh 2\xi \cos^8 \frac{\pi\xi}{2\xi_0} d\xi &= \left(\frac{35}{256} - \frac{7\xi_0^2}{32(4\xi_0^2 + \pi^2)} + \frac{7\xi_0^2}{64(\xi_0^2 + \pi^2)} \right. \\
 &\quad \left. - \frac{\xi_0^2}{8(4\xi_0^2 + 9\pi^2)} + \frac{\xi_0^2}{256(\xi_0^2 + 4\pi^2)} \right) \sinh 2\xi_0 \\
 \int_0^{2\pi} \cos 2\eta (1 + a \sin^2 \eta)^8 d\eta &= -\pi a \left(1 + \frac{a}{2} \right) \left(4 + 12a + \frac{81}{4} a^2 \right. \\
 &\quad \left. + \frac{41}{2} a^3 + \frac{407}{32} a^4 + \frac{143}{32} a^5 + \frac{715}{1024} a^6 \right) \\
 \int_0^{\xi_0} \cos^8 \frac{\pi\xi}{2\xi_0} d\xi &= \frac{35}{128} \xi_0.
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 \mu_{0,4} &= \frac{1}{4} h^2 \pi \left[\left(1 + 4a + \frac{21}{2} a^2 + \frac{35}{2} a^3 + \frac{1225}{64} a^4 + \frac{441}{32} a^5 \right. \right. \\
 &\quad \left. \left. + \frac{1617}{128} a^6 + \frac{429}{128} a^7 + \frac{6335}{32768} a^8 \right) \left(\frac{35}{64} - \frac{7\xi_0^2}{8(4\xi_0^2 + \pi^2)} + \frac{7\xi_0^2}{16(\xi_0^2 + \pi^2)} \right. \right. \\
 &\quad \left. \left. - \frac{\xi_0^2}{2(4\xi_0^2 + \pi^2)} + \frac{\xi_0^2}{64(\xi_0^2 + 4\pi^2)} \right) \sinh 2\xi_0 + \frac{35}{64} \xi_0 a \left(1 + \frac{a}{2} \right) \right]
 \end{aligned}$$

$$\cdot \left(4 + 12a + \frac{81}{4} a^2 + \frac{41}{2} a^3 + \frac{407}{32} a^4 + \frac{143}{32} a^5 + \frac{715}{1024} a^6 \right) \Bigg]. \quad (4)$$

For the second moment in elliptical coordinates,

$$\begin{aligned} \mu_2 &= \int_0^{2\pi} \int_0^{\xi_0} \left[\left(\frac{\partial F}{\partial \xi} \right)^2 + \left(\frac{\partial F}{\partial \eta} \right)^2 \right] d\xi d\eta \\ &= \int_0^{2\pi} \int_0^{\xi_0} \left[\frac{N^2 \pi^2}{4 \xi_0^2} (1 + a \sin^2 \eta)^{2N} \cos^{2(N-1)} \frac{\pi \xi}{2 \xi_0} \sin^2 \frac{\pi \xi}{2 \xi_0} \right] d\xi d\eta \\ &\quad + \int_0^{2\pi} \int_0^{\xi_0} \left[a^2 N^2 (1 + a \sin^2 \eta)^{2(N-1)} \sin^2 2\eta \cos^{2N} \frac{\pi \xi}{2 \xi_0} \right] d\xi d\eta. \end{aligned}$$

For $N=1$,

$$\begin{aligned} \mu_{2,1} &= \frac{\pi^2}{4 \xi_0^2} \left(\int_0^{2\pi} (1 + a \sin^2 \eta) d\eta \right) \left(\int_0^{\xi_0} \sin^2 \frac{\pi \xi}{2 \xi_0} d\xi \right) \\ &\quad + a^2 \left(\int_0^{2\pi} \sin^2 2\eta d\eta \right) \left(\int_0^{\xi_0} \cos^2 \frac{\pi \xi}{2 \xi_0} d\xi \right). \end{aligned}$$

Here,

$$\begin{aligned} \int_0^{2\pi} (1 + a \sin^2 \eta)^2 d\eta &= 2\pi \left(1 + a + \frac{3}{8} a^2 \right) \\ \int_0^{\xi_0} \sin^2 \frac{\pi \xi}{2 \xi_0} d\xi &= \frac{1}{2} \xi_0. \end{aligned}$$

$$\int_0^{2\pi} \sin^2 2\eta \, d\eta = \pi$$

$$\int_0^{\xi_0} \cos^2 \frac{\pi\xi}{2\xi_0} \, d\xi = \frac{1}{2} \xi_0.$$

Therefore,

$$\mu_{2,1} = \frac{\pi^3}{4\xi_0^2} \left(1 + a + \frac{3}{8} a^2\right) + \frac{\pi}{2} a^2 \xi_0. \quad (5)$$

For $N = 2$,

$$\begin{aligned} \mu_{2,2} = & \frac{\pi^2}{\xi_0^2} \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^4 \, d\eta \right) \left(\int_0^{\xi_0} \frac{1}{4} \sin^2 \frac{\pi\xi}{\xi_0} \, d\xi \right) \\ & + 4a^2 \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^2 \sin^2 2\eta \, d\eta \right) \left(\int_0^{\xi_0} \cos^4 \frac{\pi\xi}{2\xi_0} \, d\xi \right). \end{aligned}$$

Here,

$$\int_0^{2\pi} (1 + a \sin^2 \eta)^4 \, d\eta = 2\pi \left(1 + 2a + \frac{9}{4} a^2 + \frac{5}{4} a^3 + \frac{35}{128} a^4\right)$$

$$\int_0^{\xi_0} \frac{1}{4} \sin^2 \frac{\pi\xi}{\xi_0} \, d\xi = \frac{1}{8} \xi_0$$

$$\int_0^{2\pi} (1 + a \sin^2 \eta)^2 \sin^2 2\eta \, d\eta = \pi \left(1 + a + \frac{5}{16} a^2\right)$$

$$\int_0^{\xi_0} \cos^4 \frac{\pi \xi}{2\xi_0} d\xi = \frac{3}{8} \xi_0.$$

Therefore,

$$\begin{aligned} \mu_{2,2} = & \frac{\pi^3}{4\xi_0^2} \left(1 + 2a + \frac{9}{4} a^2 + \frac{5}{4} a^3 + \frac{35}{128} a^4 \right) + \frac{3}{2} \pi \xi_0 \left(1 + a \right. \\ & \left. + \frac{5}{16} a^2 \right) a^2. \end{aligned} \quad (6)$$

For $N=3$,

$$\begin{aligned} \mu_{2,3} = & \frac{9\pi^2}{4\xi_0^2} \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^6 d\eta \right) \left(\int_0^{\xi_0} \cos^4 \frac{\pi \xi}{2\xi_0} \sin^2 \frac{\pi \xi}{2\xi_0} d\xi \right) \\ & + 9a^2 \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^4 \sin^2 2\eta d\eta \right) \left(\int_0^{\xi_0} \cos^6 \frac{\pi \xi}{2\xi_0} d\xi \right). \end{aligned}$$

Here,

$$\begin{aligned} \int_0^{2\pi} (1 + a \sin^2 \eta)^6 d\eta = & 2\pi \left(1 + 3a + \frac{45}{8} a^2 + \frac{25}{4} a^3 + \frac{525}{128} a^4 \right. \\ & \left. + \frac{189}{128} a^5 + \frac{231}{1024} a^6 \right) \end{aligned}$$

$$\int_0^{\xi_0} \cos^4 \frac{\pi \xi}{2\xi_0} \sin^2 \frac{\pi \xi}{2\xi_0} d\xi = \frac{1}{16} \xi_0$$

$$\int_0^{2\pi} (1 + a \sin^2 \eta)^4 \sin^2 2\eta d\eta = \pi \left(1 + 2a + \frac{15}{8} a^2 + \frac{7}{8} a^3 + \frac{21}{32} a^4 \right)$$

$$\int_0^{\xi_0} \cos^6 \frac{\pi \xi}{2\xi_0} d\xi = \frac{5}{16} \xi_0.$$

Therefore,

$$\begin{aligned} \mu_{2,3} = & \frac{9\pi^3}{32\xi_0^2} \left(1 + 3a + \frac{45}{8}a^2 + \frac{25}{4}a^3 + \frac{525}{128}a^4 + \frac{189}{128}a^5 \right. \\ & \left. + \frac{231}{1024}a^6 \right) + \frac{45}{16}\xi_0 a^2 \pi \left(1 + 2a + \frac{15}{8}a^2 + \frac{7}{8}a^3 + \frac{21}{32}a^4 \right). \end{aligned} \quad (7)$$

For N = 4,

$$\begin{aligned} \mu_{2,4} = & \frac{4\pi^2}{\xi_0^2} \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^8 d\eta \right) \left(\int_0^{\xi_0} \cos^6 \frac{\pi \xi}{2\xi_0} \sin^2 \frac{\pi \xi}{2\xi_0} d\xi \right) \\ & + 16a^2 \left(\int_0^{2\pi} (1 + a \sin^2 \eta)^6 \sin^2 2\eta d\eta \right) \left(\int_0^{\xi_0} \cos^8 \frac{\pi \xi}{2\xi_0} d\xi \right). \end{aligned}$$

Here,

$$\begin{aligned} \int_0^{2\pi} (1 + a \sin^2 \eta)^8 d\eta = & 2\pi \left(1 + 4a + \frac{21}{2}a^2 + \frac{35}{2}a^3 + \frac{1225}{64}a^4 \right. \\ & \left. + \frac{441}{32}a^5 + \frac{1617}{128}a^6 + \frac{429}{128}a^7 + \frac{6335}{32,768}a^8 \right) \\ \int_0^{\xi_0} \cos^6 \frac{\pi \xi}{2\xi_0} \sin^2 \frac{\pi \xi}{2\xi_0} d\xi = & \frac{5}{128} \xi_0 \end{aligned}$$

$$\int_0^{2\pi} (1 + a \sin^2 \eta)^6 \sin^2 2\eta \, d\eta = \left(1 + 3a + \frac{75}{16} a^2 + \frac{35}{8} a^3 + \frac{75}{128} a^4 \right. \\ \left. + \frac{99}{128} a^5 + \frac{429}{4096} a^6 \right) \pi$$

$$\int_0^{\xi_0} \cos^8 \frac{\pi \xi}{2\xi_0} \, d\xi = \frac{55}{16} \xi_0.$$

Therefore,

$$\mu_{2,4} = \frac{5}{16} \frac{\pi^3}{\xi_0} \left(1 + 4a + \frac{21}{2} a^2 + \frac{35}{2} a^3 + \frac{1225}{64} a^4 + \frac{441}{32} a^5 \right. \\ \left. + \frac{1617}{128} a^6 + \frac{429}{128} a^7 + \frac{6335}{32,768} a^8 \right) + 55 \xi_0 \pi a^2 \left(1 + 3a + \frac{75}{16} a^2 \right. \\ \left. + \frac{35}{8} a^3 + \frac{75}{128} a^4 + \frac{99}{128} a^5 + \frac{429}{4096} a^6 \right). \quad (8)$$

III. GRAPHS

Figures 1 through 5 are plots of illuminations and far-field power patterns. All illuminations are plotted with peak amplitude equal to unity. For all far-field powers, the logarithm of the power is plotted with the center of the main lobe normalized to zero decibels.

FIGURES:

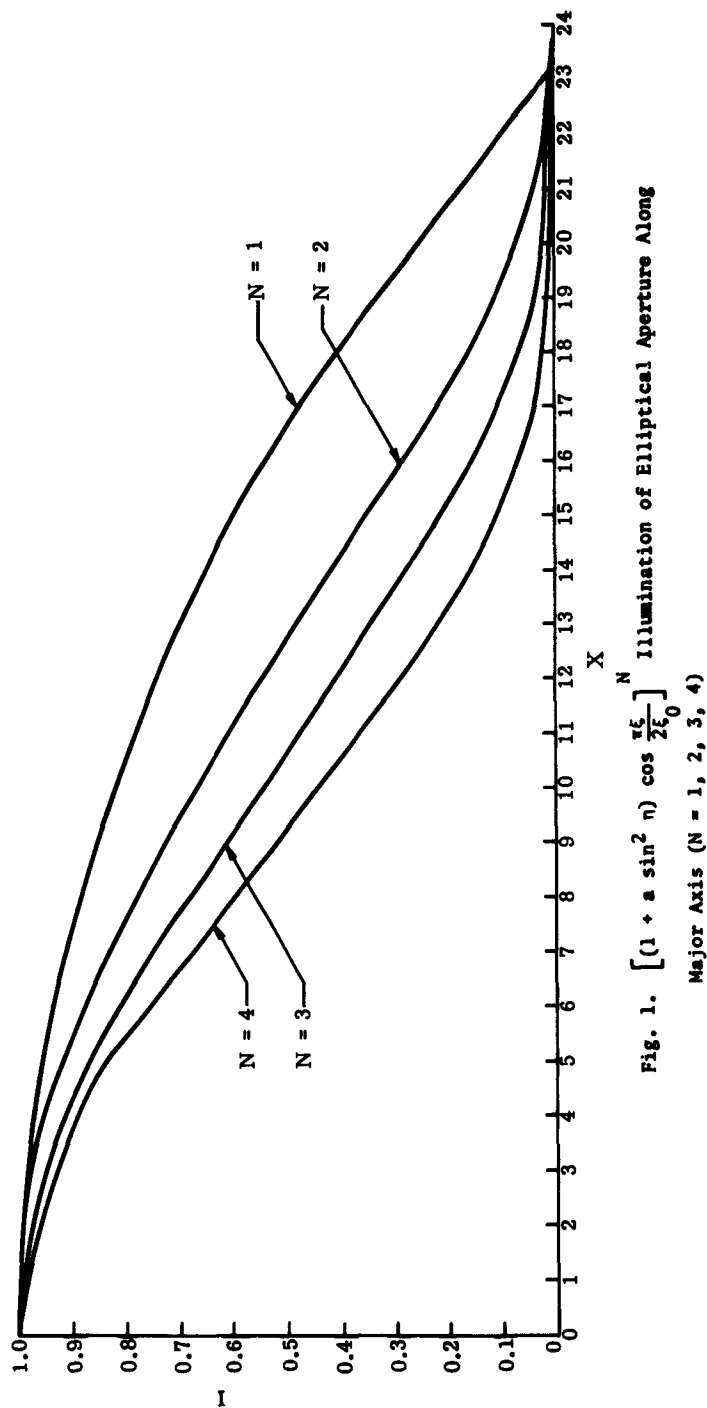
Fig. 1 = $\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N$ Illumination of Elliptical
Aperture Along Major Axis. $N = 1, 2, 3, 4$

Fig. 2 = Far-Field Power for $\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]$ Elliptical
Illumination Major Axis

Fig. 3 = Far-Field Power for $\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^2$ Elliptical
Illumination Along Major Axis.

Fig. 4 = Far-Field Power for $\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^3$ Elliptical
Illumination Along Major Axis.

Fig. 5 = Far-Field Power for $\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^4$ Elliptical
Illumination Along Major Axis.



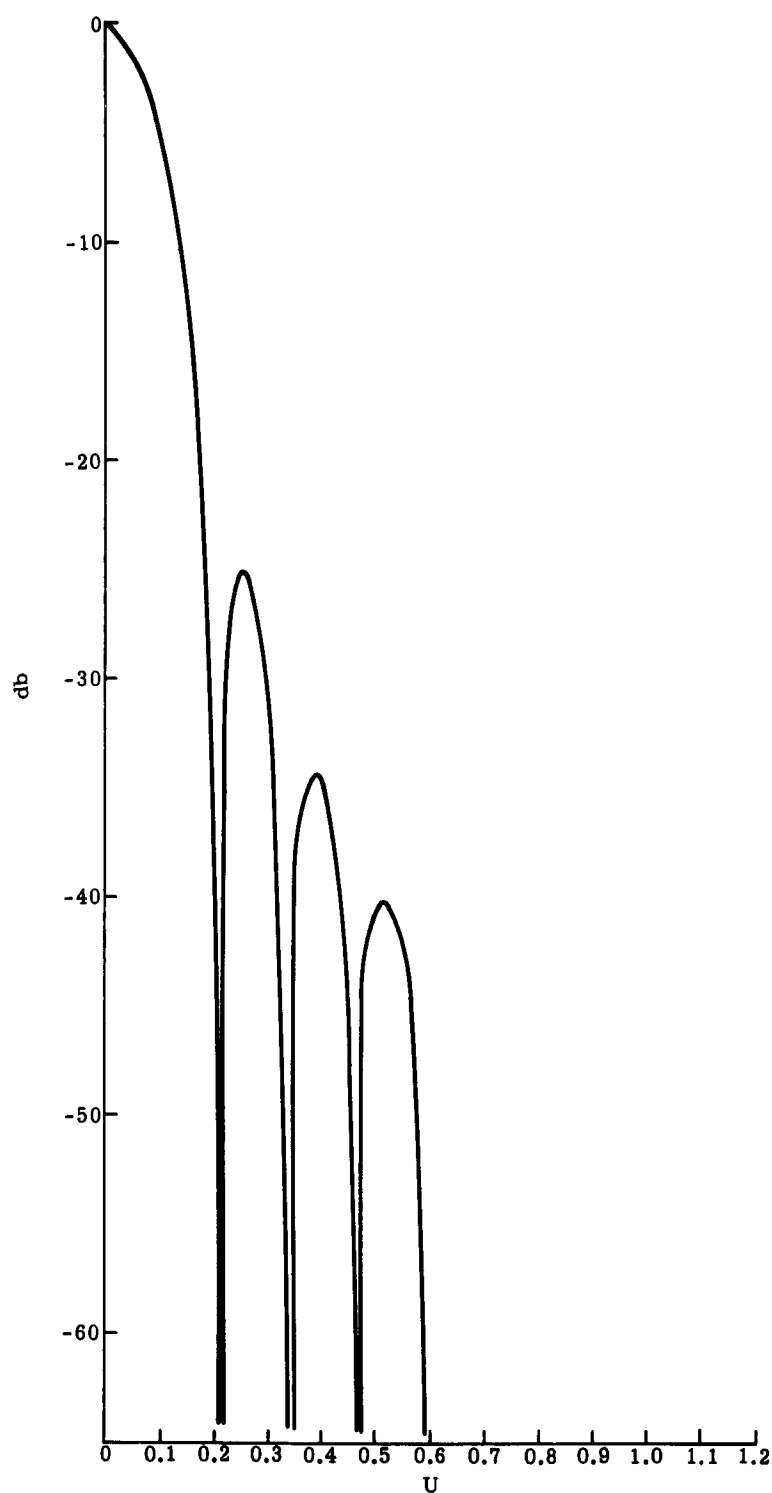


Fig. 2. Far-Field Power for $\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2 \xi_0} \right]$ Elliptical Illumination
Along Major Axis

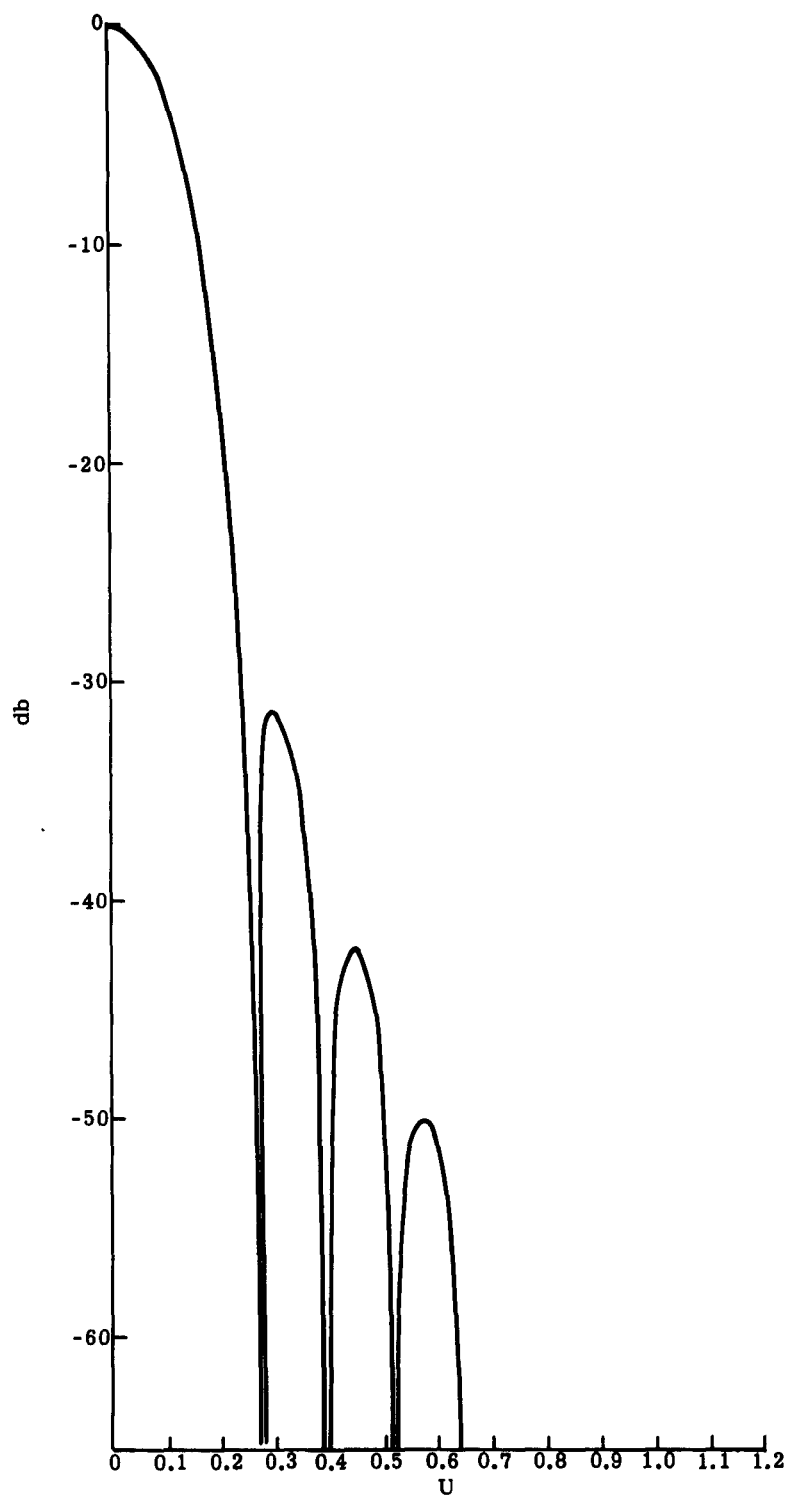


Fig. 3. Far-Field Power for $\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^2$ Elliptical Illumination
Along Major Axis

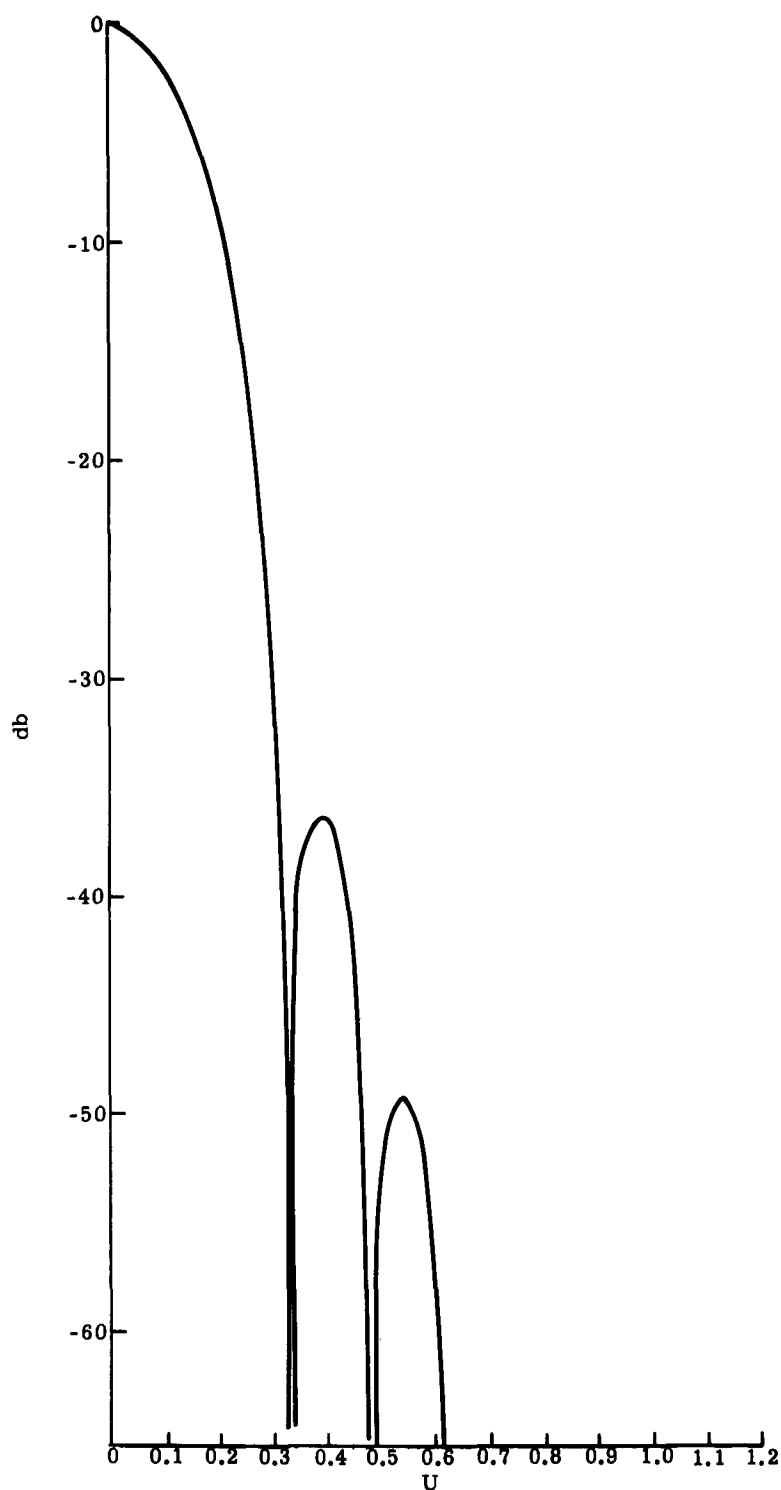


Fig. 4. Far-Field Power for $\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2 \xi_0} \right]^3$ Elliptical Illumination
Along Major Axis

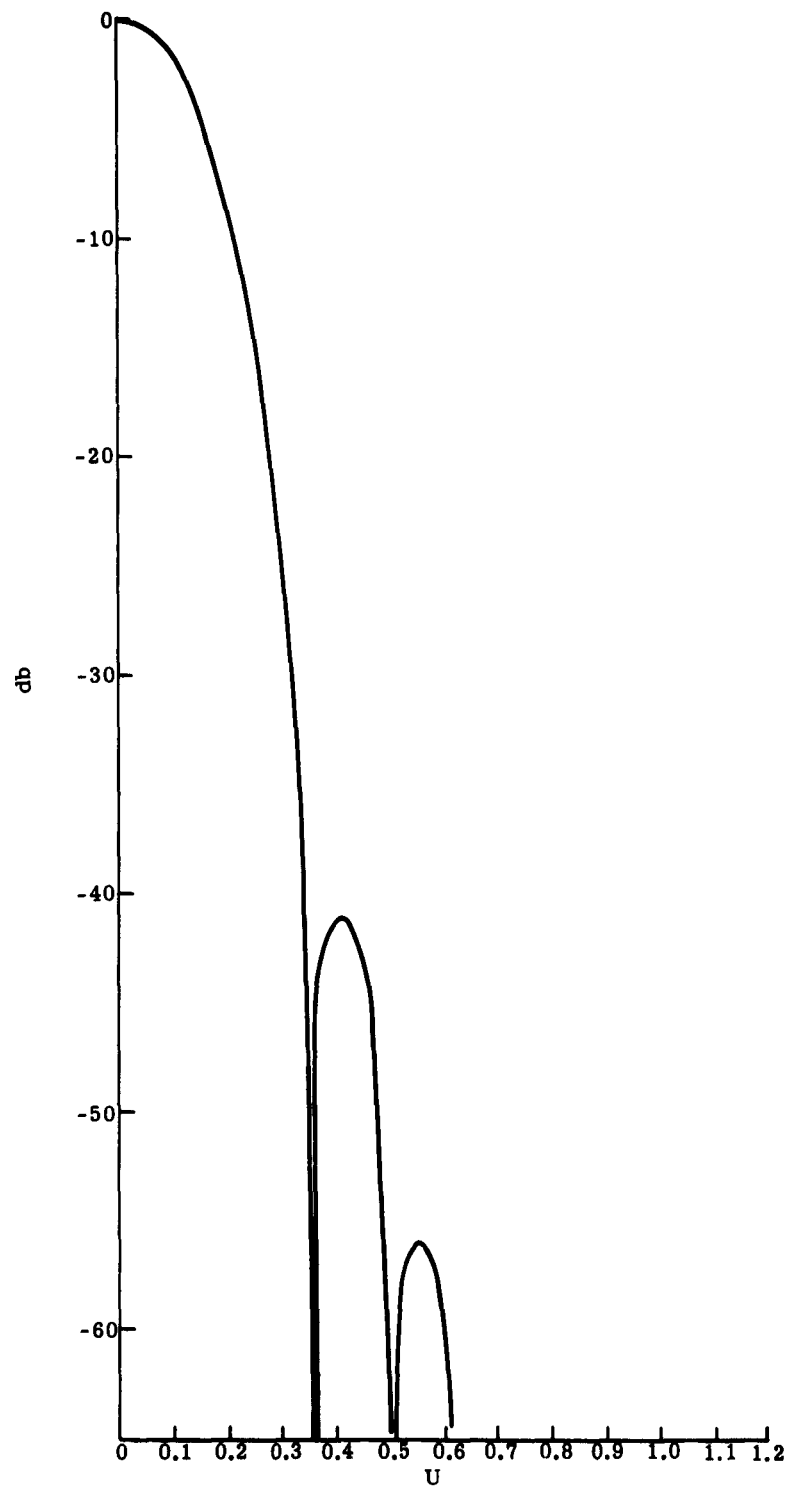


Fig. 5. Far-Field Power for $\left[\left(1 + a \sin^2 \eta \right) \cos \frac{\pi \xi}{2 \xi_0} \right]^4$ Elliptical Illumination
Along Major Axis

IV. TABLE OF COMPARISON

<u>Type</u>	<u>Illumination</u>	<u>Function</u>	<u>Beamwidth</u>	<u>Sidelobe</u>	<u>Moment</u>
Circular	Optimum	$J_0(K_{00}r)$	2.2	-28.4	5.794
Circular	Uniform	A constant	1.6	-16.8	∞
Circular	Nonoptimum	$\cos\left(\frac{\pi r}{2a}\right)$	2.1	-25.6	5.83
Circular	Nonoptimum	$\cos^2\left(\frac{\pi r}{2a}\right)$	2.3	-34	7.17
Circular	Nonoptimum	$\cos^3\left(\frac{\pi r}{2a}\right)$	2.6	-41	9.41
Elliptical	Optimum	$Ce_0(\xi, q) ce_0(\eta, q)$	0.11	-36	0.0718
Elliptical	Uniform	A constant	0.075	-17.5	∞
Elliptical	Nonoptimum	$(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0}$	0.08	-24	0.0745
Elliptical	Nonoptimum	$\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0}\right]^2$	0.095	-30	0.0962
Elliptical	Nonoptimum	$\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0}\right]^3$	0.115	-36.25	0.1455
Elliptical	Nonoptimum	$\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0}\right]^4$	0.135	-41	0.2583
Square	Optimum	$\cos \frac{\pi x}{2a} \cos \frac{\pi y}{2a}$	2.0	-22.8	4.9868

APPENDIX A
COMPUTER PROGRAMS

Program 1.

This program is designed to compute moments of the four selected nonoptimum illuminations for elliptical antennas.

$$\left(\left[(1 + a \sin^2 \eta) \cos \frac{\pi \xi}{2\xi_0} \right]^N ; N = 1, 2, 3, 4 \right)$$

The value of a is chosen to be 99.

The program calculates Eqs (1) through (8) and the moments are obtained by

$$\begin{matrix} (5) & (6) & (7) & (8) \\ \overline{\quad}, & \overline{\quad}, & \overline{\quad}, & \overline{\quad}, \\ (1) & (2) & (3) & (4) \end{matrix} , \text{ respectively.}$$

```

07300      PRINT2
07324      2 FORMAT(19H  TABLE OF MOMENTS/)
07392      AM=23.1
07416      PI=3.14159
07440      SI=0.277
07464      A=99.0
07488      R=(AM**2)*PI*A*SI*(1+.5*A)/4.
07632      S=(.1457*PI**2)/(4.*SI**2+PI**2)
07788      T=(AM**2)*PI*(1+A*(1+0.375*A))
07920      ZU01=T*S+R
07968      PRINT 22,ZU01
07992      22 FORMAT(11H  MU(0,1)=E15.8)
08044      ZU21=(PI**3/(4.*SI))*(1+A*(1+0.375*A))+1.5708*(A**2)*SI
08284      PRINT 21,ZU21
08308      21 FORMAT(11H  MU(2,1)=E15.8)
08360      V=ZU21/ZU01
08396      PRINT 1,V
08420      1 FORMAT(9H  MOMENT=E15.8///)
08482      P=PI**2
08518      Q=SI**2
08554      PA=(1+A*(2+A*(2.25+A*(1.25+(35.*A)/128.))))
08686      PSI=SI*(.375-SI*.5828*(1./(4.*Q+P)-.0625/(P+Q)))
08878      P1A=A*(2+A*(3+A*(1.875+A*0.4375)))
08986      PA1=SI*0.1875*P1A
09034      YU02=AM**2*(PI)*(PA*PSI+PA1)
09142      PRINT 31,YU02
09166      31 FORMAT(11H  MU(0,2)=E15.8)
09218      RA=1.5*PI*SI*(1+A*(1+0.3125*A))*(A**2)
09398      R1A=(0.25*(PI**3)/SI)*PA+RA
09482      PRINT 32,R1A
09506      32 FORMAT(11H  MU(2,2)=E15.8)
09558      W=R1A/YU02
09594      PRINT 1,W
09618      H=(2.5-.5828*SI*(7.5/(4.*Q+P)-.75/(P+Q)+.5/(4.*Q+9.*P)))
09918      Q=(3.+A*(7.5+A*(9.375+A*(6.5626+A*(2.38125+0.38671*A))))
10062      E=(1+A*(3.+A*(5.625+A*(6.25+A*(4.101+A*(1.476+.2255*A))))))
10230      U03=(AM**2)*PI*SI*(0.125*E*H+.15625*A*Q)
10410      PRINT 42,U03
10434      42 FORMAT(11H  MU(0,3)=E15.8)
10486      E1=(0.28125*(PI**3)/SI)*E
10558      F=(1+A*(2.+A*(1.875+A*(.875+0.65625*A))))
10678      F1=2.8125*PI*SI*(A**2)*F
10786      U23=E1+F1
10822      PRINT 41,U23
10846      41 FORMAT(11H  MU(2,3)=E15.8)
10898      O=U23/U03
10934      PRINT 1,O
10958      C=10.5
10982      D=17.5
11006      E=19.140625
11030      F=13.78125
11054      G=12.6328125
11078      H=3.3515625
11102      P=6335./32768.
11138      SA=(1.+A*(4.+A*(C+A*(D+A*(E+A*(F+A*(G+A*(H+A*P)))))))
11354      S1=7.0/(4.*SI**2+PI**2)/2.
11462      S2=0.875/(4.*SI**2+PI**2)
11558      S3=1.00/(4.*SI**2+9.*PI**2)
11678      S4=1./(32.*(SI**2+4.*PI**2))
11798      C=4.
11822      D=12.
11846      E=20.25
11870      F=20.5
11894      G=12.71875
11918      H=4.46875
11942      P=0.6982422
11966      TA=.5465*A*50.5*(C+A*(D+A*(E+A*(F+A*(G+A*(H+A*P))))))
12194      EXTRA=1.03975-.2914*SI*(S1-S2+S3-S4)
12326      U04=.25*(AM**2)*PI*SI*(SA*EXTRA+TA)
12458      PRINT 51,U04
12482      51 FORMAT(11H  MU(0,4)=E15.8)
12534      C=1.
12558      D=3.
12582      E=75./16.
12618      F=35./8.
12654      G=75./128.
12690      H=99./128.
12726      P=429./4096.
12762      RA=55.*SI*PI*(A**2)*(C+A*(D+A*(E+A*(F+A*(G+A*(H+A*P))))))
13038      TT=0.3125*(PI**3/SI)*SA+RA
13122      PRINT 52,TT
13146      52 FORMAT(11H  MU(2,4)=E15.8)
13198      T=TT/U04
13234      PRINT 1,T
13258      END

```

Program 2.

This program is divided into two parts. Part I is a program to compute far-field powers of

$$G_1(u, o) \text{ [page 9] and } G_2(u, o) \text{ [page 12]}$$

The increment of u is approximately 0.01. Part II, a similar program, computes

$$G_3(u, o) \text{ [page 15] and } G_4(u, o) \text{ [page 17]}$$

The value of a is chosen to be 99.

```

Part I
07300 3 FORMAT(18H VOLTAGE-POWER 1/)
07366 4 FORMAT(18H VOLTAGE-POWER 2/)
07432 5 FORMAT(F10.4)
07454 7 FORMAT(12H G1(0,0)=E15.8)
07508 8 FORMAT(16H LCG G1(0,0)=E15.8)
07570 10 FORMAT(4H F10.5)
07608 11 FORMAT(3H ///)
07654 12 FORMAT(10H G(0,0)=E15.8)
07704 17 FORMAT(12H G2(0,0)=E15.8)
07758 18 FORMAT(16H LCG G2(0,0)=E15.8)
07820 DIMENSION YO(60),Y1(60),Y2(60),Y3(60),Y4(60),Y5(60),Y6(60)
07820 DO 6 I=1,60
07832 6 READ 5, YO(I)
07916 DO 70 J=1,60
07928 70 READ 5, Y1(J)
08012 PRINT 3
08036 A=99
08072 P1=22./7.
08120 S1=0.277
08156 B=P1*(1+S1**2)/(16.*S1**2+P1**2)
08336 A0=49.5+101.0*P1*B
08408 A2=101.0+A*P1*B
08480 A4=9.5
08516 PRINT 7,A0
08540 IF(A0) 37,58,38
08596 37 A0=-A0
08644 38 C=LCGF(A0)
08680 PRINT 8,C
08704 I=0
08740 AX=0.0
08776 100 I=I+1
08824 AX=AX+.25
08872 Y2(I)=(2./AX)*Y1(I)-Y0(I)
09016 Y3(I)=(4./AX)*Y2(I)-Y1(I)
0916C Y4(I)=(6./AX)*Y3(I)-Y2(I)
09304 POWER=A0*YO(I)+A2*Y2(I)+A4*Y4(I)
09520 IF(POWER) 57,58,58
09576 57 POWER=-POWER
09624 58 P=LCGF(POWER)
09660 POWER=8.8658*(P-C)
09720 PRINT 10,POWER
09744 IF(I-60)100,108,108
09812 108 PRINT 11
09836 PRINT 4
09860 P=P1**2
09908 Q=S1**2
09956 B=0.5*P/(4.*Q+P)
10064 A0=(1.+A*(1+.375*A))*B+.25*A*(1+.5*A)
10268 PRINT 17,A0
10292 IF(A0) 97,98,98
10348 97 A0=-A0
10396 98 T=LCGF(A0)
10432 PRINT 18,T
10456 A2=(1+.5*A)*A*B+.25*(2.+A*(2.+7.*A/8.))
10648 A4=.125*A*(A*B+2.*(1+.5*A))
10816 A6=(A**2)/32.
10876 I=0
10912 AX=0.0
10948 200 I=I+1
10996 AX=AX+.25
11044 Y2(I)=(2./AX)*Y1(I)-Y0(I)
11188 Y3(I)=(4./AX)*Y2(I)-Y1(I)
11332 Y4(I)=(6./AX)*Y3(I)-Y2(I)
11476 Y5(I)=(8./AX)*Y4(I)-Y3(I)
11620 Y6(I)=(10./AX)*Y5(I)-Y4(I)
11764 POWER=A0*YO(I)+A2*Y2(I)+A4*Y4(I)+A6*Y6(I)
12052 IF(POWER) 87,88,88
12108 87 POWER=-POWER
12156 88 P=LCGF(POWER)
12192 POWER=8.8658*(P-T)
12252 PRINT 10,POWER
12276 IF(I-60)200,208,208
12344 208 PAUSE
12356 END

```

Part II

```

07300 5 FORMAT(18H VOLTAGE-POWER 3/)
07366 6 FORMAT(18H VOLTAGE-POWER 4/)
07432 13 FORMAT(13H LN G(0,0)=E15.8)
07488 15 FORMAT(19H F10.5/)
07560 16 FORMAT(11H G4(0,0)=E15.8)
07612 17 FORMAT(F10.4)
07634 DIMENSION Y0(60),Y1(60),Y2(60),Y3(60),Y4(60),Y5(60),Y6(60)
07634 DIMENSION Y7(60),Y8(60),Y9(60),Y10(60)
07634 A=99.
07670 P1=22./7.
07718 S1=.277
07754 P=P1**2
07802 Q=S1**2
07850 DO 1 I=1,60
07862 1 READ 17, YQ(I)
07946 DO 2 J=1,60
07958 2 READ 17, YI(J)
08042 PRINT 5
08066 B1=1./((16.*Q+P)
08138 B2=1./((16.*Q+9.*P)
08246 B=B1-B2
08294 PCNE=1.+A*(1.5+A*(1.125+A*0.3125))
08402 PTWC=A*(1.5+A*(1.5+15.*A/32.))
08410 SQ=1.+(.277*.277)
08470 A0=1.5*P*SQ*B*PCNE+2.*PTWC/3.
08714 A2=1.5*P*SQ*B*PTWC+2.*(2.+A*(3.+A*(21./8.+13.*A/16.)))/3.
08978 A4=.5625*P*SQ*50.5*(A**2)*B+2.*A*(1.5+A*(1.5+.5*A))/3.
09254 A6=(3./64.)*P*SQ*(A**3)*B+.25*50.5*A*A
09458 A8=A**3/48.
09518 A10=0.0
09554 IF(A0) 37,38,38
09610 37 A0=A0
09658 38 POWER=LOGF(A0)
09694 PRINT 13,POWER
09718 J=0
09754 200 I=0
09790 AX=0.0
09826 J=J+1
09874 100 I=I+1
09922 AX=AX+.25
09970 Y2(I)=(2./AX)*Y1(I)-Y0(I)
10114 Y3(I)=(4./AX)*Y2(I)-Y1(I)
10258 Y4(I)=(6./AX)*Y3(I)-Y2(I)
10402 Y5(I)=(8./AX)*Y4(I)-Y3(I)
10546 Y6(I)=(10./AX)*Y5(I)-Y4(I)
10690 Y7(I)=(12./AX)*Y6(I)-Y5(I)
10834 Y8(I)=(14./AX)*Y7(I)-Y6(I)
10978 Y9(I)=(16./AX)*Y8(I)-Y7(I)
11122 Y10(I)=(18./AX)*Y9(I)-Y8(I)
11266 G=A0*Y0(I)+A2*Y2(I)+A4*Y4(I)+A6*Y6(I)+A8*Y8(I)+A10*Y10(I)
11698 IF(G) 57,58,58
11754 57 G=G
11802 58 GG=LOGF(G)
11838 R=8.6858*(GG-POWER)
11898 PRINT 15,R
11922 IF(I-60) 100,206,206
11990 206 IF(J-2) 207,208,208
12058 208 PAUSE
12070 207 PRINT 6
12094 SS=(.277*.277)*(2./((4.*Q+P)-.125/(P+0)))
12286 C=35./128.
12334 TTT=(1.+A*(2.+A*(2.25+A*(1.25+C*A))))*(.375-SS)
12514 A0=TTT+.1875*A*(2.+A*(3.+A*(1.875+7.*A/16.)))
12694 PRINT 16,A0
12718 IF(A0) 67,68,68
12774 67 A0=A0
12822 68 POWER=LOGF(A0)
12858 PRINT 13,POWER
12882 C=5.25
12918 D=3.25
12954 E=49./64.
13002 T=0.1375*(2.+A*(4.+A*(C+A*(D+A*E))))
13146 A2= A*(2.+A*(3.+A*(1.875+0.4375*A)))*(0.375-SS)+T
13326 C=7./32.
13374 R=(A**2)*(0.75+A*(0.75+A*C))
13506 T=0.1875*A*(2.+A*(3.+A*(2.+0.5*A)))
13662 A4=R*(0.375-SS)+T
13734 R= 0.125*(A**3)*(1.+0.5*A)
13854 T=(A*A)*0.1875*(0.75+A*(0.75+29.*A/128.))
14010 A6=R*(0.375-SS)+T
14082 A8=(A**4/128.)*(0.375-SS)+(3./128.)*A**3*(1.+0.5*A)
14246 A10=3.*A**4/2048.
14418 GO TO 200
14426 END

```